

Midterm Practice II
Math 183, UCSD, Fall 2017
(Do not turn in!)

Exercise I

You are the head of the Food and Drug Administration (F.D.A.), in charge of deciding whether new drugs are effective and should be allowed to be sold to people. A pharmaceutical company trying to win approval for a new drug they manufacture claims that their drug is better than the standard drug at curing a certain disease. The company bases this claim on a study in which they gave their drug to 1000 volunteers with the disease. They compared these volunteers to a group of 1000 hospital patients who were treated with the standard drug and whose information is obtained from existing hospital records. The company found a "statistically significant" difference between the percentage of volunteers who were cured and the percentage of the comparison group who were cured. That is, they did a statistical hypothesis test and rejected the null hypothesis that the percentages are equal. As director of the F.D.A., should you permit the new drug to be sold? Explain your reasoning in three or less sentences.

Exercise II

By now it is well known that Target Corporation (Target) "knew a teen girl was pregnant before her father did". Not only was the story told many times over in the New York Times, but it also became one of the lead examples illustrating the intrinsic value of "big data". A bit creepy, yes, but basically Target uses a pregnancy-prediction score, inferred from past purchases, to develop a pregnancy-likelihood and confidence interval on every woman who shops at Target. They use this score to target baby product ads at the right time and to the right people.

If you were a statistician at the Target corporation, and you were given the data of size 10,000 with an average of pregnancy-prediction score of 0.6 and standard deviation of 0.04, can you provide a 95% confidence interval for one single women possibly entering the Target store?

Exercise III

The distribution of U.S. adult women is nicely modeled by a normal distribution. You are wondering about actual the mean weight of U.S. adult females. You sample 30 randomly picked adult female's weights, which yields $\bar{x} = 146$ pounds, with a sample standard deviation $s_x = 15$ pounds.

1. Build a 80% confidence interval for the mean of U.S. adult females
2. You read on a website that the mean weight of U.S. adult females is equal to 140 pounds. Using $\alpha = 0.05$, conduct a hypothesis test to see if this info is true. Make sure to: 1) state hypotheses, 2) draw a picture of the sampling distribution, 3) shade the p -value, 4) find the p -value, and 5) make a decision about the hypotheses based on the p -value.

Exercise IV

Let X be a continuous random variable such that,

$$P(X \leq x) = \begin{cases} 0 & \text{if } x < 0 \\ x^5 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1. \end{cases}$$

1. What is the density function $f_X(x)$ of the random variable X ?
2. Compute the expected value and the variance of X .
3. Let $Y = X + 2$. Compute the density function $f_Y(y)$ of Y .

(Hint: You may start with computing $P(Y \leq y)$)

Exercise V

In this exercise, for $x \in \mathbb{R}$, we denote by $\lceil x \rceil$ the integer approaching x from above. For instance, $\lceil 1.2 \rceil = 2$, $\lceil 1 \rceil = 1$, and $\lceil \pi \rceil = 4$.

Let X be a random variable with distribution $Exp(\lambda)$.

1. Compute, for all integer $k \geq 1$, the probability that X is between $k - 1$ and k .
2. Deduce that $Y = \lceil X \rceil$ has a geometric distribution $Geom(p)$, with parameter p to be specified.

Exercise VI

Your roommate is very clumsy, to the extent that he has a 30% chance to let any dishes he touches fall. This includes your favorite coffee cup you use every morning. Unfortunately, this week, it's his turn to do the washing up, so he'll do it 7 times.

1. What is the probability that he breaks your favorite coffee cup during the week?
2. He is aware of his clumsiness, so he bought a tea cup that can resist four falls. What is the probability his cup is not broken at the end of the week?